Quiz 6 Solutions — 12 October 2018 Name:

Question 1 (2 pts)**.** Mark each statement true or false, and give a brief justification of your answer.

(a) If $A, B \in M_{n\times n}(F)$ are similar, then $\det(A) = \det(B)$.

True, because $\det(B) = \det(QAQ^{-1}) = \det(Q)\det(A)\det(Q)^{-1} = \det(A)$.

(b) If $A \in M_{n\times n}(F)$ is a matrix and $k \in F$ is a scalar, then $\det(kA) = k\det(A)$.

False; what's true is that $\det(kA) = k^n \det(A)$.

Question 2 (4 pts)**.** Compute the determinant of the following matrix.

$$\begin{pmatrix} 2 & 3 & 3 \\ 0 & 3 & -1 \\ 2 & 0 & 5 \end{pmatrix}$$

There are many ways to compute the determinant, but here's one. Recall that type 3 row operations do not change the determinant. Subtracting the first row from the third, and then adding the second row to the third, we get

$$\begin{pmatrix} 2 & 3 & 3 \\ 0 & 3 & -1 \\ 2 & 0 & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 3 & 3 \\ 0 & 3 & -1 \\ 0 & -3 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & 3 & 3 \\ 0 & 3 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

The last matrix is upper triangular, so its determinant (which is equal to the determinant of our original matrix) is the product of the diagonal entries, which is 6.

Question 3 (4 pts)**.** Prove that if $A \in M_{n\times n}(\mathbb{R})$ with n odd and $\det(-A) = \det(A)$, then A is not invertible.

Note that $-A = -I \cdot A$ and $\det(-I) = (-1)^n = -1$, because $-I$ is diagonal (in particular, upper triangular) with diagonal entries n copies of -1, and the product of these is -1 because n is odd. Now using multiplicaticity of the determinant, we get

$$\det(-A) = \det(-I \cdot A) = \det(-I)\det(-A) = -\det(A).$$

But we're told that $\det(-A) = \det(A)$, so $\det(A) = -\det(A)$, which implies $\det(A) = 0$, and this is equivalent to A not being invertible.